Graphical fuzzy algebra applied to schedulability analysis of real time systems

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Abstract
This paper proposes a graphical representation for the computation of complex functions with two fuzzy arguments. It is based on the theory of evidence, and has been compared with the theory of falling shadows with similar results for the AND operation. It is graphically shown that no general correspondence between grades of membership correlation and t-norms can be established when the extension principle is applied. Some new interpretations of membership correlation, based on imprecise observations, are provided. The proposed method has been used to solve real-time schedulability analysis with fuzzy task execution times. The example shows that interval computations can be performed to reduce the number of time consuming simulations.

Keywords: Belief theory, possibility theory, extension principle, fuzzy algebra, schedulability analysis, rate monotonic algorithm.

1 Introduction
This paper proposes a graphical representation for the computation of complex functions with two fuzzy arguments. Using the theory of evidence [1], the function and the focal elements of the underlying consonant body of evidence of each of its arguments are obtained and represented in the same graph, allowing an alternative calculation of the extension principle. The rest of the paper is organised as follows.

Section 2 reviews some basic definitions.

Sections 3 and 4 describe the proposed graphical method applied to the computation of joint possibility/plausibility distribution and to the extension principle. Same conclusions as in [2], [3] are reached.

In section 5 the concept of joint imprecise observations [4] is reviewed and an additional interpretation for the correlation of grades of membership is proposed. This may suggest new criteria to choose between both ways of extending real functions to fuzzy arguments.

Section 6 applies the precedent results to an example of real time scheduling computations with fuzzy execution times. Ending times for each task are calculated and compared with their maximum allowed times (deadlines). Approximate results can be obtained using interval analysis, when the extension principle is applied with the min t-norm, with a drastic reduction of the number of simulations. Moreover the possibility or necessity that a task finishes before its deadline can be exactly computed.

Conclusions and future work are detailed in section 7.

2 Definitions review
Plausibility measure is a fuzzy measure \( P : P(X) \rightarrow [0,1] \) such that [5],[1]:

\[
\text{Pl}(A_i \cap A_j \cap \ldots A_n) \leq \sum_i \text{Pl}(A_i) - \sum_{i<j} \text{Pl}(A_i \cup A_j) + \ldots + (-1)^{n+1} \text{Pl}(A_i \cup A_j \cup \ldots A_n)
\]

where \( P(X) \) is the power set of crisp subsets of \( X \), being \( X \) the universe of discourse.

Given a body of evidence \((F,m)\), a plausibility measure can also be defined as:

\[ P(B) = m(A) \]

where \( A_i \) are the focal elements and \( m \) is the basic probability assignment [1].

When the focal elements are nested, the body of evidence is said to be consonant, and the plausibility measure is called a possibility measure. In this case the following property holds:

\[ P(A \cup B) = \text{max}\{P(A), P(B)\} \rightarrow \mathcal{P}(A \cup B) = \text{max}\{\mathcal{P}(A), \mathcal{P}(B)\} \]

Given a body of evidence \((F,m)\) the contour function \( P(\{x\}) \) can be interpreted as a fuzzy set membership function \( \mu(x) \) [6]:

\[
\mu(x) = P(\{x\}) = \sum_{A_i \cap \{x\} \neq \emptyset} m(A_i) = \sum_{x \in A_i} m(A_i)
\]
In addition, the body of evidence is consonant, then the plausibility distribution function \( P_l((a,b)) \) is a possibility distribution function \( \pi((x,y)) \), denoted by \( \pi(x) \). In this context the membership function \( \mu(x) \) can also be interpreted as a possibility distribution function of the focal elements, and Montecarlo method can be used to obtain a set \( A \) and perform numerical simulations [7],[8].

The membership function \( \mu(a) : \mathbb{R} \rightarrow [0,1] \) of a fuzzy set \( A \) can also be represented in terms of its \( \alpha \)-cuts [2],[9]:

\[
\mu(a) = \sup \{ \alpha \in (0,1) / \alpha \in A_a \}
\]

where \( A_a \) are the \( \alpha \)-cuts of \( A \), that is: \( A_a = \{ a / \mu(a) \geq \alpha \} \).

In this context, denoting by \( A_i \) the \( \alpha \)-cut of level \( \alpha_i \), or \( \alpha \)-cut, then \( A_i \) can be interpreted as a focal element of the underlying consonant body of evidence of \( A_i \). \( (F_{A_i},m_{A_i}) \) are the \( \alpha \)-cuts [7],[2], where: \( m_{A_i}(A) = \alpha \alpha_i \) (by convention \( \alpha_i \alpha_i > \alpha \alpha_i > ... \), \( \alpha_i \alpha_i > \alpha \alpha_i \) and \( \alpha_i \alpha_i = 0 \)).

Let’s suppose that a fuzzy set \( A \) is constructed from a set of consonant imprecise observations \( A_i \). Then, a body of evidence \( (F_{A_i},m_{A_i}) \) can be obtained from the set of observations where \( m_{A_i}(A) \) is the probability of the crisp set \( A_i \). \( (F_{A_i},m_{A_i}) \), and a mapping can be established between \( A_i \) and \( A_i \) (in the sequel the level \( \alpha \) of the \( \alpha \)-cut \( A_i \) will be denoted by \( A_i \)).

When two imprecise observations \( A_i \) and \( B_j \) are performed simultaneously, a joint basic assignment can be calculated \( (\alpha_i \alpha_j) \) and \( \alpha_i \alpha_j \) of the \( \alpha \)-cuts \( A_i \) and \( B_j \) respectively [3].

Some different basic assignments may be considered [3],[10]:

- The basic assignment is uniform in the main diagonal of the unit square \([0,1]^2\); or \( \alpha_i = \alpha_j \): this is called perfect positive correlation.
- The basic assignment is uniform on the unit square \([0,1]^2\); this corresponds to \( \alpha_i \) and \( \alpha_j \) independence.
- The basic assignment is uniform in the anti-diagonal of the unit square \([0,1]^2\); or \( \alpha_i = 1 - \alpha_j \), called perfect negative correlation.

### 3 Joint possibility/plausibility distribution

When the sources are completely reliable, [11], [12], the joint possibility/plausibility distribution of \( A \) and \( B \) is obtained by conjunctive consensus.

Under possibility theory, the available information relating \( A \) \( (\pi(a) : \mathbb{R} \rightarrow [0,1]) \) and \( B \) \( (\pi(b) : \mathbb{R} \rightarrow [0,1]) \) can be taken into account by selecting the appropriate t-norm to combine the possibility distributions of \( A \) and \( B \).

The joint possibility distribution \( \pi_{\alpha_{A,B}} \) is given by:

\[
\pi_{\alpha_{A,B}}((a,b)) = T(\pi_A(a),\pi_B(b))
\]

where \( a \in \mathbb{R} \), \( b \in \mathbb{R} \) and \( T \) is a t-norm. From now on, \( \mathbb{R} \) will be the universe of discourse.

Under the theory of evidence, this information can be modelled by the joint basic assignment of \( \alpha_{A_1} \) and \( \alpha_{A_2} \) (see section 3.1). Both alternatives do not necessarily express the same relationship between \( A \) and \( B \), although the joint possibility distributions obtained with \( \min \) product and Lukasiewicz t-norms are the same ones as the plausibility distributions obtained with \( \alpha_{A_1} = \alpha_{A_2} \), independent alphas and \( \alpha_{A_1} = 1 - \alpha_{A_2} \) respectively [3].

Next section describes a new graphical method to obtain the joint plausibility distribution of \( A \) and \( B \), which is compared with possibility theory.

### 3.1 Joint distribution in theory of evidence

Given two fuzzy numbers \( A \) and \( B \), with underlying consonant bodies of evidence \( (F_{A_i},m_{A_i}) \) and \( (F_{B_i},m_{B_i}) \), the joint plausibility of any pair \((a,b)\) can be obtained by [3]:

- Supposing an existing relationship between \( \alpha_{A_1} \) and \( \alpha_{A_2} \) expressed by means of a joint basic assignment \( m_{A_{1,2}}(\alpha_{A_1},\alpha_{A_2}) \) defined over a subset of \([0,1]^2\).
- Mapping each pair \((\alpha_{A_1},\alpha_{A_2})\) to a pair \((a,b)\).
- The relationship between \( \alpha_{A_1} \) and \( \alpha_{A_2} \) define the set of valid pairs \((a,b)\). The joint focal elements \( C_i \) are the cartesian product of the pairs \((a,b)\).
- Computing the joint basic probability assignment \( m_C \) as the probability \( P \) of every pair \((\alpha_{A_1},\alpha_{A_2})\):

\[
m_C = m_{A_{1,2}}(\alpha_{A_1},\alpha_{A_2})
\]

The joint body of evidence will be denoted by \( (F_{A_{1,2}},m_{A_{1,2}}) \).

- Computing the plausibility of any pair \((a,b)\) as:

\[
P_l((a,b)) = m_{A_{1,2}}(C_i)
\]

If the joint body of evidence is consonant, this plausibility measure is also a possibility measure.

In the sequel, for the sake of simplicity, fuzzy numbers are supposed to be triangular. Simpler graphs will be obtained although the conclusions are valid for any other shape. Symmetry is not assumed in this analysis.

The proposed method consists of representing the lower limits \( l(A_i) \) and \( l(B_i) \) of the focal elements \( A_i \) and \( B_i \) in the \( x \) and \( y \) axis, instead of their corresponding \( \alpha_{A_1} \) and \( \alpha_{A_2} \) levels. Each pair \((l(A_i),l(B_i))\) represents a joint focal element \( C_i \). The domain of the lower limits of the \( \alpha \)-cuts of a fuzzy number \( A \) will be denoted by \( \bigcup_{A_1} \). When \( A \) is a triangular fuzzy number \( l(A_i) \) is uniformly distributed in
The joint basic assignment of $\alpha_A$ and $\alpha_B$ is concentrated on the main diagonal of $[0,1]^2$, which means that $(l(A_j),l(B_k))$ is also uniformly distributed on the main diagonal of lower limits domain $U_{l(A)} \times U_{l(B)}$.

Figure 1 shows that in this case the joint focal elements are nested, and thus plausibility measures are possibility measures. Joint focal elements are Cartesian products $C_i = A_j \times B_k$ and have been drawn in the full universe $\mathcal{A} \times \mathcal{B}$ (being $\mathcal{A}$ the support of $\mathcal{A}$).

In the discrete case, the possibility of a point $(a_1,b_1)$ (see figure 1) is calculated summing the mass assignment of every joint focal element containing the pair $(a_1,b_1)$:

$$\pi((a_1,b_1)) = \sum_{i=1}^n m_{c,a,b}(C_i)$$

This point belongs to the line defined by $\alpha_A = \alpha_B$, and thus $\pi((a_1,b_1)) = \pi_A(a_1) = \pi_B(b_1)$. Additionally, since the joint focal elements containing $(a_2,b_2)$ are the same as those containing $(a_1,b_1)$, then it is:

$$\pi((a_2,b_2)) = \pi((a_1,b_1)) = \pi_B(b_2) = \min(\pi_A(a_2),\pi_B(b_2))$$

The same joint possibility distributions are obtained using belief theory from the one hand, and possibility theory with the $\min$ t-norm from the other hand.

### 3.1.2 Independence relationship

When $\alpha_A$ and $\alpha_B$ are uniformly distributed on the unit square $[0,1]^2$, we say they are independent. In this case $(l(A_j),l(B_k))$ is uniformly distributed on the whole lower limits domain $U_{l(A)} \times U_{l(B)}$.

In figure 4.b a lower limits domain representation is shown highlighting the focal elements to be considered in the plausibility calculus. In figure 2, four joint focal elements $C_1, C_2, C_3$ and $C_4$ are shown: since they are not nested, the plausibility measures associated to the joint body of evidence are not possibility measures. In this figure, joint focal elements are shown as Cartesian products $C_i = A_j \times B_k$ and drawn in the full universe $\mathcal{A} \times \mathcal{B}$.

In this case the plausibility of $(a_i,b_j)$ is given by:

$$P((a_i,b_j)) = \pi_A(a_i) \cdot \pi_B(b_j)$$

and thus the plausibility distribution obtained from the theory of evidence is the same as the possibility distribution obtained from the possibility theory, using the product t-norm, even if the joint body of evidence is not consonant [2].

### 3.1.3 Perfect negative correlation

When $\alpha_A$ and $\alpha_B$ are uniformly distributed on the anti-diagonal of the unit square $[0,1]^2$, $(l(A_j),l(B_k))$ is also uniformly distributed on the anti-diagonal of lower limits domain $U_{l(A)} \times U_{l(B)}$.

In this case (see figure 3), the focal elements are not nested, giving a non consonant body of evidence, and thus, plausibility measures are not possibility measures. Figure 4.c shows $(l(A_j),l(B_k))$ points representing joint focal elements that have to be considered in $P((a_i,b_j))$ (or
In figure 3, \( P((a_1,b_1)) = 0 \) since no joint focal element contains it. To calculate \( Pl((a_2,b_2)) \), every joint focal element from \( C_i \) to \( C_k \) have to be considered. In this figure, joint focal elements are shown as Cartesian products \( C_i \times B_j \) drawn in the full universe \( A \times B \).

For any pair \((a,b)\), it is:

\[
Pl((a,b)) = \max\left(0, \pi_a(a) + \pi_b(b) - 1\right)
\]


4 Extension principle revisited

Two alternatives can be used to extend real functions to fuzzy arguments. Possibility theory establishes the following extension principle [11]:

\[
\pi_c(c) = \sup_{c \in [a,b]} (\pi(a,b)) = \sup_{\alpha \in \alpha(a,b)} (T(\pi_a(\alpha), \pi_b(\alpha)))
\]

The second alternative is to apply belief theory as described in section 3.1, where a joint body of evidence must be calculated. Let’s suppose for simplicity that each value of \( c \) can be obtained from two pairs \((a_1,b_1)\) or \((a_2,b_2)\) (see figure 4). The possibility of any set is the maximum possibility of every point belonging to it, that is:

\[
\pi((a_2,b_2)) = \sup\left(\pi((a_2,b_2))\right)
\]

Under belief theory, given the body of evidence \((F_{a \times B}, m_{a \times B})\), the plausibility measure of a two-point set is given by:

\[
Pl((a_2,b_2)) = \max_{\alpha \in \alpha(a_2,b_2)} (\sup(C_i))
\]

that is, every joint focal element containing at least one of the two points must be considered.

Figure 4.a shows the joint focal elements that contain \((a_2,b_2)\) when \(\alpha = \alpha_0\). In this case, the plausibility of the union is equal to the plausibility of \((a_2,b_2)\), which is the maximum plausibility of both \(\max\) operator is obtained when focal elements are nested, [1]). The result is the same as in possibility theory because the underlying body of evidence is also the same.

Figure 4.b shows the joint focal elements when \(\alpha_i\) and \(\alpha_0\) are independent. All the joint focal elements located in the shadowed area contain at least one of the two points and must be considered. For continuous variables, the plausibility is the integral of the uniform distribution \(m_{a \times B}\) over the shadowed area, that is the volume whose base is the shadowed area. Since this volume is in general greater or equal than the volume obtained for \(Pl((a_2,b_2))\), or \(Pl((a_2,b_2))\), then it is:

\[
Pl((a_2,b_2)) = \max\left(\max(Pl((a_2,b_2)), Pl((a_2,b_2)))\right)
\]

Figure 4.c shows the joint focal elements that contain at least \((a_2,b_2)\) or \((a_2,b_2)\) when \(\alpha = 1-\alpha_0\). For continuous variables, its plausibility is given by the area with thick line base, and in general it is greater or equal than the individual plausibility measures.

The correspondence between t-norms in possibility theory and alphas correlation (reviewed in section 3) does not generally holds [2]. Possibility theory always considers, among all the different bodies of evidence with the same possibility/plausibility distribution, the underlying consonant one (see figure 5). Several studies have been performed to the extension principle for the case of fuzzy arithmetic with different t-norms [13],[14],[15].
**Independence**: Different experts give intervals $A_i$ and $B_i$ for $a$ and $b$ variables (using independent criteria), or $a$ and $b$ are measured with different instruments, with independent measurement principles.

**Perfect negative correlation**: In this case, there is a total amount of accuracy that must be shared between $a$ and $b$ observations. For example, a sole expert located between two objects is estimating their heights: the nearer the expert is from the first object, the more precise will be the first object height, but the less precise will be the second one. In the same way, if two measurements are performed with a total fixed number of bits, the more precise is the first measurement (more bits used), the less precise is the second one.

### 6 Application to real time scheduling

Real-time applications have to respect some time constraints, and consists of a number of tasks competing to the same CPU resources. Tasks can be described by their [16]:

- **Deadline**: tasks must finish before their deadline.
- **Ready time**: tasks cannot start before their ready time.
- **Period**: each task is periodically executed, its ready time and deadline being within each period.
- **Execution time**.

The scheduler has to dynamically decide which task should be executed. When more than one task are ready, the one with the highest priority will get the control of the CPU. Tasks can be pre-emptive or non pre-emptive. In the pre-emptive scheme there is always an immediate switch to the highest priority ready task [17]. Priorities can be determined before the execution (statically), or during the execution, depending on the changing situation (dynamically). Different scheduling algorithms can be defined [17], some of them being:

- **Rate monotonic scheduling (RMS)**: static priorities are defined according to the size of the period: the task with the shortest period has the highest priority.
• deadline monotonic scheduling: static priorities are assigned according to the deadlines: the shorter the deadline, the higher the priority
• earliest deadline first: dynamic priorities are defined according to the time distance between the actual time and the nearest deadline: the task with the nearest deadline has the highest priority

Depending on the scheduling algorithm, the system will or will not be able to respect its deadlines.

In several simple cases, equations have been formulated to calculate the schedulability of a system, such as earliest deadline algorithm when deadlines coincide with periods. In most cases there is no exact equation, and the system must be simulated in order to verify that it is schedulable. It is enough to simulate for the time of the longest period to assure schedulability when all the tasks are released at the same instant, called critical instant [18].

Many schedulability analysis using crisp task descriptions have been proposed. There exist also some considering imprecision or uncertainty in execution times [19], but they all apply to simple cases based on explicit equations.

However it is interesting to get information about the schedulability of the system from the very beginning of the design process: the earlier the problem is found, the cheaper will be its correction. Since in these early phases of development designers have just an approximate idea about the tasks, their knowledge could be modelled in terms of fuzzy sets. Fuzzy set theory can give more information than the simpler “worst case”, consisting in considering the greatest execution times of every task.

This section shows how the schedulability of a system can be analysed with simulations performed using fuzzy execution times. We will only consider periodic tasks, and is then extended to the fuzzy case.

Possibility distributions are obtained for the ending execution times of each task at each iteration. They are then compared to the deadlines, giving the possibility/necessity that a task finishes before its deadline. The example starts with crisp execution times to the tasks ending times at every iteration. In this case, RMS: \( \mathbb{R}^1 \rightarrow \mathbb{R}^9 \). The algorithm has been previously described.

Let us consider now the fuzzy execution times of task \( A \) exactly at 12, the possibility that \( A \) ends between 13 and 15 drops vertically to 0. Since ending time of task \( A \) is 14, the possibility that \( A \) finishes at 13. This adds up to non convex fuzzy set \( F \).

The real time example consists of three periodic tasks \( A \), \( B \) and \( C \) with periods: 3, 5 and 15 respectively. Crisps execution times are \( e_A=1 \), \( e_B=2 \) and \( e_C=3 \). With RMS algorithm, task \( A \) has the highest priority while \( C \) the lowest.

Figure 6 shows a graph of the schedule of the application called time-line [17]. The three tasks are released at the critical instant.

Looking at \( A \) ending times, any deadline between 1 and 3 (its period) can be respected. Looking at \( B \) ending times, if its deadline is between 3 and 5 it can be respected. \( C \) task deadline should be between 14 and 15.

RMS algorithm is a non continuous function defined from the tasks execution times, to the tasks ending times at each iteration. In this case, RMS: \( \mathbb{R}^3 \rightarrow \mathbb{R}^9 \). The algorithm has been previously described.

Let us consider now the fuzzy execution times of task \( B \) at its third execution. Extension principle can be applied to obtain the fuzzy value \( F \) of the variable \( f \) when fuzzy execution times are considered:

\[
\mu_f(f) = \sup_{(e_A,e_B,e_C)} \left( \min \left( \mu_{e_A}(e_A), \mu_{e_B}(e_B), \mu_{e_C}(e_C) \right) \right)
\]

where \( rms \) function is the RMS algorithm having as sole output \( B \)-task-ending-time at its third iteration.

\( F \) can be computed by sampling \( E_A \), \( E_B \) and \( E_C \) supports with \( n_A \), \( n_B \) and \( n_C \) numbers of samples, and simulating every \( (e_A,e_B,e_C) \) combination to obtain the possibility distribution of \( F \). This adds up to \( n_A \times n_B \times n_C \) simulations.

Figure 7 shows the resulting non convex fuzzy set \( F \).

The gap is owed to the fact that, when \( A \) task is executing, \( B \) can neither execute nor finish. Since \( A \) task is released exactly at 12, the possibility that \( B \) finishes at 12 drops to 0. Since ending time of task \( A \) at its fifth execution is not crisp, possibility distribution of \( B \) ending time at its third execution, \( F \), grows linearly from 12.9 to 13.
Figure 8 shows several lines $f = \text{rms}(e_A, e_B) = \text{constant} \in [11.9, 12] \cup [12.9, 13.3]$. The joint focal elements $R_i$ and $R_a$, which are nested and located along the diagonal $\alpha_{e_i} = \alpha_{e_a}$ are also shown. It can be seen that since task $C$ has the lowest priority, it does not interfere in the execution of $B$, and $f$ only depends on the combination of $A$ and $B$ execution times.

An approximation of $F$ can be obtained by interval analysis:

$$\left[ F, \overline{F} \right](\alpha) = \text{rms}(E_{A\epsilon}, E_{B\epsilon}), \text{rms}(\overline{E}_{A\epsilon}, \overline{E}_{B\epsilon})$$

where $\left[ F, \overline{F} \right](\alpha)$ is the interval representation in terms of lower and upper limits of the $\alpha$-cut of $F$ of level $\alpha$ [20].

The pairs $(e_A, e_B)$ located on the main diagonal of the lower limits domain, $[0.9,1] \times [1.9,2]$, are the lower limits of $A$ and $B$ $\alpha$-cuts such that $\alpha_{e_A} = \alpha_{e_B}$.

Extending this to the upper limits, the pairs $(e_A, e_B)$ on the main diagonal of the upper limits domain $[1.1,1] \times [2.2,1]$ are the upper limits of $A$ and $B$ $\alpha$-cuts such that $\alpha_{e_A} = \alpha_{e_B}$.

$F$ grows from 11.9 to 12, where there is a discontinuity, the following value 12.9 being at (0.9, 2). It continues growing until 13.3 located at (1.1, 2.1). To compute for example the possibility of 11.95, the basic assignment of all joint focal elements from $R_a$ to $R_i$ have to be summed because all of them contain at least one point of the line $f = 11.95$. This possibility coincides with the possibility of the pair (0.95, 1.95) located at the main diagonal of the lower limits domain:

$$\pi(11.95) = \pi_{e_A}(0.95) = \pi_{e_B}(1.95) = \pi(13.15) = \pi_{e_A}(1.05) = \pi_{e_B}(2.05) = 0.5.$$

11.95, 0.95 and 1.95 are the lower limits of $F$, $E_A$ and $E_B$ $\alpha$-cuts with $\alpha = 0.5$. 13.15, 1.05 and 2.05 are the upper limits of $F$, $E_A$ and $E_B$ $\alpha$-cuts with $\alpha = 0.5$. This process can be extended to all the lower and upper limits of the $\alpha$-cuts, finally leading to:

$$\left[ F, \overline{F} \right](\alpha) = \text{rms}(E_{A\epsilon}, E_{B\epsilon}), \text{rms}(\overline{E}_{A\epsilon}, \overline{E}_{B\epsilon})$$

obtaining the possibility $\alpha$ of every $f \in [11.9, 12] \cup [12.9, 13.3]$.

These results show that it is not necessary to simulate every combination of $(e_A, e_B, e_C)$ to compute lower and upper limits of $F$ possibility distribution, just lower and upper limits with same $\alpha$. But a wrong possibility of 1 is obtained for ending times between 12 and 13.

It should be noted that to compare ending times with deadlines, this approximation is still valid:

- If the deadline $d$ is after 13.3, the possibility that $B$ finishes before its deadline is $\Pi = 1$, and the necessity is $N = 1$.
- If the deadline $d$ is between 13 and 13.3 then $\Pi = 1$ and $N = 1 - \mu_d(d)$
- If deadline is between 12 and 13: $\Pi = 1$, $N = 0$
- If deadline $d$ is between 11.9 and 12: $\Pi = \mu_d(d)$, $N = 0$
- If deadline is before 11.9: $\Pi = 0$, $N = 0$

To compare the number of simulations applying the extension principle and applying interval analysis, it can be considered an $\alpha$ resolution of $r$. (0.1) is divided into $l/r$ intervals, giving $n_A = \frac{2}{r} - 1 = n_B = n_C = N$.

Number of tasks $= N^3$ simulations are necessary to apply the extension principle while only $N$ simulations are necessary to apply interval analysis.

Furthermore, if the objective is to find the deadline respected with a possibility $\Pi_k$, only the simulation $\text{rms}(E_{A\epsilon}, E_{B\epsilon}, E_{C\epsilon})(\alpha)$ with $\alpha = \Pi_k$ must be performed. In the same way, to find the deadline that would be respected with a necessity $N_1$, only $\text{rms}(E_{A\epsilon}, E_{B\epsilon}, E_{C\epsilon})(\alpha)$ with $\alpha = 1 - N_1$ must be simulated.
It is currently being proved that interval calculus can be always applied to schedulability analysis when $\alpha_s = \alpha_{s'}$ is used, not only with RMS algorithm but with other scheduling algorithms, and that the result coincides with the possibility extension principle with min t-norm.

7 Conclusions

A graphical method to operate fuzzy numbers has been proposed, and compared with other similar techniques, and some well known results have been graphically reviewed. With this method plausibility measures can be directly obtained from the same graph where the variables are represented: no translation from variables domain to $\alpha$ domain must be done. This can be specially useful for complex functions with fuzzy arguments.

The method has been applied to schedulability analysis of real-time systems with RMS (rate monotonic scheduling) algorithm, using a simulation based approach with fuzzy inputs. Traditional RMS has been extended using the extension principle with min t-norm. It is graphically shown that in this case interval analysis can be applied to perform an exact schedulability analysis, considerably reducing the number of simulations. However, the possibility distributions of the ending times obtained are just approximate.

Other real-time scheduling algorithms are being graphically analysed and simulated, and we expect similar results when execution times are fuzzy numbers and min t-norm (that is $\alpha_a = \alpha_b$ correlation) is used. Other t-norms and other $\alpha$ relationship are also being considered. Fuzzy modelling of task period and deadlines is also being investigated.

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