Application of sensitivity analysis in investment project evaluation under uncertainty and risk

Petar Jovanović
Faculty of Organizational Sciences, 154 Jove Ilica, Belgrade 11000, Yugoslavia

This paper treats investment decision-making under uncertainty and risk. Some of the methods used for investment decision-making under uncertainty and risk are presented: Break-even Analysis, Sensitivity Analysis, Theory of Games and Decision Making Theory. Sensitivity Analysis is given special consideration, and one of the procedures of its application in investment decision making under uncertainty and risk is shown.

Introduction
The real world we are living in is a world of uncertainty, a world whose future occurrences and conditions we are, in most cases, not able to predict. Permanent confrontation of Man with this growing complexity, together with the need to overcome it, force one to continually forecast future circumstances of Nature in order to get adapted. One is compelled to predict because he/she needs to take appropriate management action in this confrontation with the environment. The only clearly defined certainty is the past, while investment problems relate to future only.

The majority of management-related problems of an enterprise, including management of investments, are lived through under uncertainty, with absence of a priori information necessary for solutions thereof. The lack of any possibility to predict future events and parameters largely affects correct evaluation of investment projects and decreases the realistic possibilities of investment decision-making.

It is quite clear that investment decision-making never takes place under conditions of certainty, but only under those of uncertainty or risk. It is therefore necessary to define and locate the investment decision-making problem in its real conditions, and possibly find suitable and appropriate solutions. Certainty used when applying quantitative criteria and methods is only the price of decreasing the great complexity in order to make it possible for us to use exact instruments and means. It is quite obvious, however, that we are living in a world of uncertainty, whose environment conditions we are mostly not able to predict perfectly. This impossibility to foresee potential future events accurately enough, and predict practical values needed for exact preparation of decisions, affects investment decision-making to a large measure and decrease possibilities of choice. We still do not have appropriate quantitative methods today which could provide us with optimal investment decisions in the circumstances of uncertainty. However, better knowledge of decision-making process in cases of uncertainty, and of decision-making criteria offered by theory as well, surely exerts influence by improving choices and by minimizing the possibility of decision-making.

Investment decision making in cases of uncertainty
Solutions of investment-related problems under conditions of uncertainty, especially evaluation of investment projects in conditions of uncertainty and risk, are possible to perform applying different methods and techniques. The best known methods employed in investment decision making are:

- Break-even Analysis
- Sensitivity Analysis
- Scenario Method
- Theory of Games and Decision Making Theory, etc.

Break-even Analysis is one of the elementary and rather simple methods used to analyse problems of investment project evaluation under conditions of uncertainty. The Break-even point of an investment project is the level of production and sales where the project produces neither income nor losses; the level which still makes the financial result positive.

The Break-even level is the boundary between the profits and losses. Above this point the project incurs losses. Break-even
can be expressed as the critical point of production (critical use of production capacity), or as the critical sale income (critical sale price per unit). Break-even Analysis is a static approach to investment project evaluation in cases of uncertainty, because it uses data on only one representative year of the project life cycle.

Application of the Break-even Analysis in investment project evaluation in cases of uncertainty is based on the idea of the critical values of certain parameters which exert a great influence on the total profitability of the project. In other words, this means calculation and analysis of the critical or the minimum values of the production scope and income from sale below which the project becomes unacceptable, as well as taking steps to avoid such. Break-even Analysis is a very simple method, with lots of weaknesses. It should therefore be used only for the initial analysis in investment project evaluation in cases of uncertainty.

For investment decision-making in cases of uncertainty, and for mathematical modelling of the decision making principles, the Theory of Games and Decision Making Theory are also used. The Theory of Games provides mathematical models of conflict situations, and with the assistance of certain principles provides solutions of such situations. In order to solve a management-related problem, and of course a problem of evaluation and choice of investments using the Theory of Games, it is necessary to deploy it in its matrix form. Matrix games used for solution of conflicts and uncertain situations can be: games against the Intelligent Opponent, and the games against the Nature.

In a game against an Intelligent Opponent, we suppose that the adversary will behave in an intelligent and rational manner, and that he/she will choose the best possible strategy to maximize his/her potential income or minimize his/her potential loss. We thus try to predict the opponent’s strategy and to create our own, using such predictions. That makes the choice in such games adjusted to the opponent.

In the games against Nature, our opponent is Nature, so we cannot be certain about any rational behaviour. In such situations we take that behaviour to be uncertain. While a game against the Intelligent Opponent makes our choice determined by predicting possible behaviour of the adversary, in a game against Nature we presume that no reliable information about possible behaviour is provided, and therefore our choice is free.

Solution of investment problems, i.e. investment decision-making, belongs to the category of games against Nature. The differing and varying future conditions which should be predicted, and which are shown through several varying size values relevant for a choice can be more or less known, and thus the degree of their uncertainty can be higher or lower.

The usual classification used in investment decision-making comprises different degrees of knowledge about the future:

- uncertainty
- risk
- certainty

Investment decision-making under conditions of uncertainty is the decision-making where we predict several different values of criteria for each investment alternative, and where we do not know which alternative will be realized in practice. Investment decision-making in cases of risk is the decision-making where we predict several different values of criteria for each alternative, but with the known probability of their occurrence. Investment decision-making in cases of certainty is the decision-making where each alternative always gives one and the same criterion value, i.e., where we are able to predict all necessary future values absolutely correctly.

A rather large number of criteria and methods are applied by the Theory of Games and Decision Making Theory used for a solution to the problems of investment decision-making under conditions of uncertainty. Among the best known are: Minimax Criterion, Maximax Criterion, Hurwicz’s Criterion, Laplace’s Criterion, Savage’s Criterion, etc.

### Sensitivity Analysis

Investment project evaluation, and research of criteria which make the basis of this evaluation are performed through an agency of certain input values serving for calculation of individual criteria. Due to the effect of different factors it is potentially possible that these input values are not realized in the future, which makes our final evaluation scores incorrect. If we want to take into consideration all possible consequences, we have to analyse, in advance, the effect of potential changes of the starting values on the final factual state or results obtained by the calculation with these values, which is performed through procedures of the Sensitivity Analysis.

Sensitivity Analysis is the calculating procedure used for prediction of effect of changes of input data on output results of one model. This procedure is often used in investment decision making related with the investment project evaluation under conditions of uncertainty.

Sensitivity Analysis of the effectiveness criterion for investment project evaluation is the calculating procedure of researching and determining the effect of changes of individual values taken into the calculation on the values of individual criteria, as well as on the final investment project evaluation. In other words, it is a procedure that analyses how the changes of certain input values (income, costs, value of investments, etc.), produced due to inappropriate prediction or for some other reason, influence certain criteria values and the total investment project evaluation. Applying this analysis it is possible to find the maximum or minimum points which one value may take while, however, still allowing an investment project to be justified and acceptable for realization.

In the investment project evaluation we have at our disposal a set of criteria (Net Present Value, Internal Rate of Return, Pay-back Period, etc.) as the basis for evaluation (set of output values), and the set of values (income, costs, discount rate, value of investments, etc.) on the basis of which we can calculate certain individual criteria (input values), as shown by the diagram in Figure 1.
**Net Present Value**

The Net Present Value criterion is defined as a sum of present values of annual net incomes earned in the period of the project exploitation.\(^5\) Mathematical expression of this criterion is:

\[
\text{NPV} = \sum_{k=0}^{n} \frac{NI_k}{(1+i)^k}
\]

where the symbols stand for:

- \(NI_k\): net incomes in the \(k\)th year of the period of project exploitation
- \(i\): discount rate
- \(n\): period of the project exploitation

**Internal Rate of Return**

The Internal Rate of Return is the Discount Rate at which the Net Present Value Criterion is zero. Mathematical expression is the following:

\[
\text{NPV} = \sum_{k=0}^{n} \frac{NI_k}{(1+i)^k} = 0
\]

**Pay-back Period**

The Pay-back Period is defined as the period (years) for which the discounted net income per year will cover the discounted total value of investments. Mathematical expression of this criterion is:

\[
\sum_{k=0}^{n} \frac{I_k}{(1+i)^k} = \sum_{k=0}^{n} \frac{NI_k^*}{(1+i)^k}
\]

where the symbols stand for:

- \(I_k^*\): value of investments in the \(k\)th year
- \(NI_k^*\): net income of the project in the \(k\)th year of the exploitation period, disregarding the value of the investments.

Generally, Sensitivity Analysis of the criteria for investment project evaluation can be presented in the following way. If some input value, e.g. total value of investment \(I\), ranges in the interval \(I_{m-1}\) to \(I_{m+1}\), what is the interval of movement of the required criteria? And next, what is the minimum value \(I\) can take, to make the investment project still proper and acceptable for realization in accordance with the chosen criterion?

The basic purpose of Sensitivity Analysis is not only to get an insight into the impact of changes of different parameters on changes of certain criteria values, but to understand the impact of such changes on the total evaluation of a certain investment project validity. Another goal is to define steps and actions of purposeful influence to be exerted on certain factors in order to avoid possible unwanted changes of some input values and of investment project evaluation.\(^6\)

If it is discovered by Sensitivity Analysis that, for instance, increase of the realized total value of investment must not exceed 50% of the starting value or the investment project would not be profitable, undertaking measures to stay within this limit in the investment planning and realization is obviously necessary. If transgression over the limit is possible, in accordance with predictions, to be higher than the maximum allowed (due to the effect of different actual and subjective factors), certain changes in the investment project itself are needed in order to increase its efficiency and to reduce its sensitivity on increase of the investment value.

A methodological approach to Sensitivity Analysis of the criteria for investment project evaluation can be presented, in quite general terms, in the following way. First, we define a set of quantitative criteria which will serve as the basis for the investment project evaluation. Defined after that is a set of input values observed in calculation of criteria, and we select the values whose influence will be analysed, e.g. income from the investment project (\(P\)), investment value (\(I\)), discount rate (\(i\)), etc. Then we determine the range over which these values may vary (\(I_{m-1}\) to \(I_{m+1}\)), to be used for calculation of individual criteria values. Calculated after that are the values of individual criteria in order to define the values of certain input variables to determine the maximum and minimum values that certain variables can take with the investment project still remaining profitable, as well as to present the obtained results. Finally, we analyse and interpret the results, and determine the measures and actions that would help us to possibly prevent or remove adverse impacts and make certain improvements.\(^2\)

It is of course possible to set procedures for conduct of the Sensitivity Analysis in some other ways, depending on concrete conditions and the defined target of the analysis. It can be very interesting, for instance, to research and study the influences of all relevant decisive factors determining one criterion, with the help of Sensitivity Analysis. We can, for example, search for influences and effects of certain values (exploitation period, investment period, discount rate, revenue per individual years, total value of investment, etc.) on the Net Present Value criterion. In this way we collect important elements for analysis of the relevant criterion, and for determination of its advantages and shortcomings, in other words—for determination of its convenience for the investment project evaluation.
Sensitivity Analysis of criteria for investment project evaluation is a very complex calculating procedure, which can only be performed with the help of a computer. Numbers of criteria considered and of input values may be very large, so that it would make calculations without the computer difficult, long-lasting, and unreasonable.

Application of Sensitivity Analysis in investment decision-making

Concrete application of Sensitivity Analysis in investment decision-making under the conditions of uncertainty involves several key input parameters, such as: incomes, costs, value of investments, discount rate, etc., as well as consideration of influences and effects of changes of these parameters on the values of several basic criteria which serve for the investment decision making, such as: the criterion of Net Present Value, the criterion of Internal Rate of Return, and the criterion of Pay-back Period. Application of Sensitivity Analysis by employing these criteria will be presented here.2

Net incomes can be presented in the following way:

\[ NI = P - T - I \]

where the symbols stand for:

- \( P \) projects incomes
- \( T \) project costs
- \( I \) value of investments

From the equation above it is possible to analyse which key factors are of the decisive influence on the criteria of the Net Present Value, Internal Rate of Return and Pay-back Period. These factors are the project incomes and costs, total value of investment, discount rate, period of the project exploitation, etc. These parameters represent input values for calculation of the Net Present Value, Internal Rate of Return, and Pay-back Period, and we have to predict their values in this calculation. It is quite clear, of course, that these parameters in future can assume values totally different from the predicted ones. In order to see and analyse all of these possible situations in the future during the investment decision-making, we vary the input parameters by attributing the relating corrective coefficients, as expressed in the formula:

\[ NI = d \cdot P - m \cdot T - k \cdot I \]

where the symbols stand for:

- \( d \) income correction factor
- \( m \) cost correction factor
- \( k \) the investment value correction factor

We will also observe the discount rate correction factor, marking it as "\( p \)".

This formula shows that the Net Present Value criterion is the function of the corrective factors presented above. The procedure of application of the Sensitivity Analysis, conducted with use of appropriate software, can be presented as in Table 1.

In this case, we analyse the simultaneous effects of discount rate and the total value of investment on the Net Present Value criterion. This means that we vary the factors \( p \) and \( k \), while factors \( d \) and \( m \) remain constant. Through this analysis we can obtain a spectrum of values of the Net Present Value criterion (NPV) serving the function of discount rate and total investment value.

From Table 1 we can see and conclude that, for example, for the discount rate \( p_1 \) a transgression of the investment value limit of 5% is allowed (\( k_1 = 1.05 \)). For the same value of the discount rate a transgression of the investment value limit of 10% (\( k_2 = 1.10 \)) gives a negative and unacceptable value of the Net Present Value criterion. It is obvious that with the Sensitivity Analysis it is possible to, as we have shown before, involve and conduct analysis of any combination of input parameters, in this manner obtaining a great deal of information about the effect of input parameters on the examined criteria for investment decision-making. With this procedure we decrease the uncertainty of parameters which will appear in practice in the future, and improve investment decision-making under conditions of uncertainty.

As an illustration of this application of Sensitivity Analysis for investment decision-making under conditions of uncertainty, we offer an example of results of the Sensitivity Analysis of one real investment project, presented in a table form. The example we are presenting relates to a metal processing industry enterprise, and considers investments into modernization and enlargement of production capacities. Calculations of the necessary investment established that the total investment value was \( I = $1,751,587 \). Discount rate used in the calculations was \( i = 15\% \). Table 2 and Table 3 present some results of Sensitivity Analysis for the Net Present Value, Internal Rate of Return, and Pay-back Period criteria.

All combinations which bring \( NPV < 0 \) are unacceptable; on the other hand, all combinations produ-
cing NPV > 0 are acceptable. In Table 2 there are two separate zones, bordered off by the dotted line. The zone situated below the line yields unacceptable results, while the zone above the line offers acceptable results. This means that with 20% for the discount rate, and 26% for increase of the total value of investment, we have NPV < 0, and our investment project is unacceptable. At the same time, for the starting total value of investment, with the discount rate of up to 28.2% the Net Present Value criterion is negative. The discount rate (28.2%) is the value of the Internal Rate of Return criterion. The graph of the Net Present Value criterion dependence on the discount rate is given in Figure 2.

### Conclusion

This paper studies the investment project evaluation and the decision-making problems under the conditions of uncertainty. Evaluation of investment projects under uncertainty and risk is possible to be carried out through application of various methods and techniques. The best known methods are: Break-

---

Table 2 Net Present Value criterion

<table>
<thead>
<tr>
<th>p/k</th>
<th>1.0</th>
<th>1.26</th>
<th>1.51</th>
<th>1.77</th>
<th>2.03</th>
<th>2.29</th>
<th>2.54</th>
<th>2.80</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00%</td>
<td>3008309</td>
<td>2557901</td>
<td>2107493</td>
<td>1657085</td>
<td>1206677</td>
<td>756269</td>
<td>305860</td>
<td>-144548</td>
</tr>
<tr>
<td>2.00%</td>
<td>2559173</td>
<td>1722794</td>
<td>1272386</td>
<td>821978</td>
<td>757541</td>
<td>307133</td>
<td>-143275</td>
<td>-593683</td>
</tr>
<tr>
<td>4.00%</td>
<td>2173202</td>
<td>1722794</td>
<td>1272386</td>
<td>821978</td>
<td>371570</td>
<td>-78838</td>
<td>-529246</td>
<td>-979654</td>
</tr>
<tr>
<td>6.00%</td>
<td>1839850</td>
<td>1389442</td>
<td>939034</td>
<td>486626</td>
<td>-251083</td>
<td>-412190</td>
<td>-862598</td>
<td>-1313007</td>
</tr>
<tr>
<td>8.00%</td>
<td>1550549</td>
<td>1100141</td>
<td>649733</td>
<td>199325</td>
<td>-251083</td>
<td>-70491</td>
<td>-1151900</td>
<td>-1602308</td>
</tr>
<tr>
<td>10.00%</td>
<td>1298303</td>
<td>847894</td>
<td>397486</td>
<td>-52922</td>
<td>-533300</td>
<td>-953738</td>
<td>-1404146</td>
<td>-1854554</td>
</tr>
<tr>
<td>12.00%</td>
<td>1077370</td>
<td>626962</td>
<td>175554</td>
<td>-273854</td>
<td>-72426</td>
<td>-1174670</td>
<td>-1625078</td>
<td>-2075486</td>
</tr>
<tr>
<td>14.00%</td>
<td>883020</td>
<td>432612</td>
<td>-17796</td>
<td>-468204</td>
<td>-918612</td>
<td>-1369020</td>
<td>-1819429</td>
<td>-2269837</td>
</tr>
<tr>
<td>16.00%</td>
<td>711333</td>
<td>260925</td>
<td>-189483</td>
<td>-639891</td>
<td>-1090299</td>
<td>-1540707</td>
<td>-1991115</td>
<td>-2441524</td>
</tr>
<tr>
<td>18.00%</td>
<td>559050</td>
<td>108642</td>
<td>-341766</td>
<td>-792174</td>
<td>-1242582</td>
<td>-1692990</td>
<td>-2143398</td>
<td>-2593806</td>
</tr>
<tr>
<td>20.00%</td>
<td>423450</td>
<td>-26958</td>
<td>-77367</td>
<td>-927775</td>
<td>-1378183</td>
<td>-1828591</td>
<td>-2278999</td>
<td>-2729407</td>
</tr>
<tr>
<td>22.00%</td>
<td>302247</td>
<td>-148161</td>
<td>-598569</td>
<td>-1048777</td>
<td>-1499385</td>
<td>-1949793</td>
<td>-2400201</td>
<td>-2850609</td>
</tr>
<tr>
<td>24.00%</td>
<td>193521</td>
<td>-256887</td>
<td>-707295</td>
<td>-1157703</td>
<td>-1608111</td>
<td>-2058519</td>
<td>-2508927</td>
<td>-2959335</td>
</tr>
<tr>
<td>26.00%</td>
<td>95645</td>
<td>-354763</td>
<td>-805171</td>
<td>-1255579</td>
<td>-1705987</td>
<td>-2156395</td>
<td>-2606803</td>
<td>-3057211</td>
</tr>
<tr>
<td>28.00%</td>
<td>7240</td>
<td>-443168</td>
<td>-895376</td>
<td>-1343984</td>
<td>-1794392</td>
<td>-2244800</td>
<td>-2695208</td>
<td>-3145616</td>
</tr>
<tr>
<td>30.00%</td>
<td>-72869</td>
<td>-523277</td>
<td>-973685</td>
<td>-1424093</td>
<td>-1874501</td>
<td>-2324909</td>
<td>-2775317</td>
<td>-3225725</td>
</tr>
</tbody>
</table>

Table 3 Internal Rate of Return and Pay-back Period

<table>
<thead>
<tr>
<th>k</th>
<th>1.00</th>
<th>1.26</th>
<th>1.51</th>
<th>1.77</th>
<th>2.03</th>
<th>2.29</th>
<th>2.54</th>
<th>2.80</th>
</tr>
</thead>
<tbody>
<tr>
<td>IRR (%)</td>
<td>28.2</td>
<td>19.6</td>
<td>13.8</td>
<td>9.5</td>
<td>6.2</td>
<td>3.5</td>
<td>1.3</td>
<td>No</td>
</tr>
<tr>
<td>PBP (years)</td>
<td>4.8</td>
<td>7.5</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
</tbody>
</table>

Figure 2 Net Present Value criterion dependence on discount rate
even Analysis, Sensitivity Analysis, Scenario Method, Theory of Games and Decision Making Theory, etc. Sensitivity Analysis of criteria for investment project evaluation is a very complex procedure. Applications of the Sensitivity Analysis by using the Net Present Value, Internal Rate of Return, and Pay-back Period criteria have been presented. With the Sensitivity Analysis we are able to obtain a great deal of information about the effect of input parameters on the examined criteria for investment project decision making. We can also collect information about the influence of input parameters on the criteria values and thus improve investment decision-making.

References